Lesson 23: A Summary of Curve Sketching

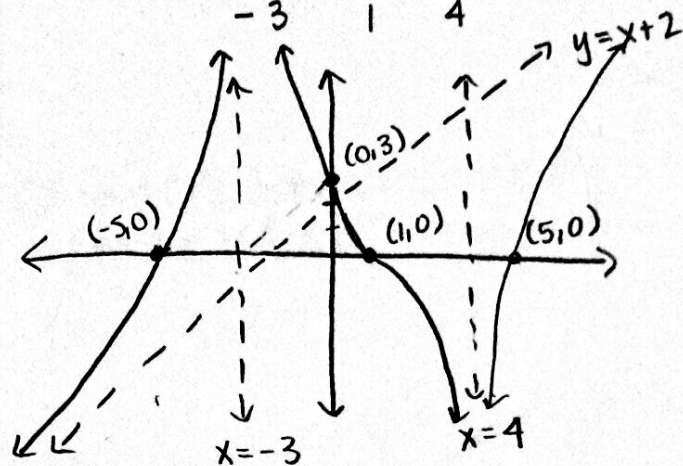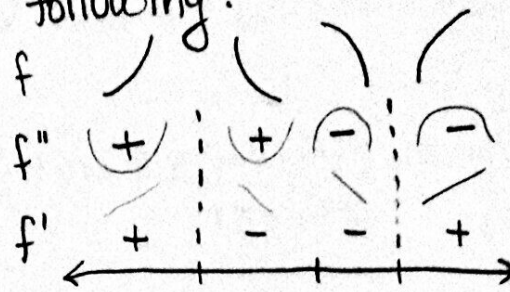Ex 2   Sketch a graph satisfying the following:

VA's: $x = -3, x = 4$

SA : $y = x + 2$

x-ints: $(-5, 0), (1, 0), (5, 0)$

y-int: $(0, 3)$

f

f" $(+)$ $(+)$ $(-)$ $(-)$

f' $+$ $-$ $-$ $+$

-3 1 4

$y = x + 2$

$(0, 3)$

$(-5, 0)$ $(1, 0)$ $(5, 0)$

$x = -3$ $x = 4$

Ex 1

$$y = \frac{x}{x^2-4}$$

① $x^2 - 4 \neq 0$

$x \neq -2, \; x \neq 2$

domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

② y-int: $(0,0)$ $(x=0)$

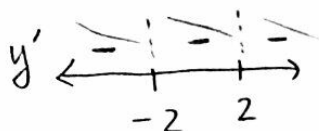x-int: $(0,0)$ $(y=0)$

$0 = \frac{x}{x^2-4} \rightarrow x = 0$

HA: $y = 0$ (warmup)

VA: $x = -2, \; x = 2$ $(x^2-4=0)$

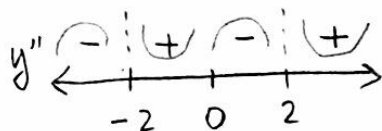③ $y' = \frac{-(x^2+4)}{(x^2-4)^2}$

CV's: $x^2 + 4 = 0$ $x^2 - 4 = 0$

no CV's   ~~$x = -2, x = 2$~~ not in domain

$y' \xleftarrow{\;\; - \; \vdots \; - \; \vdots \; - \;\;}$

      -2 2

decr: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

④ $y'' = \frac{2x(x^2+12)}{(x^2-4)^3}$

Possible IPs: $x = 0$, ~~$x = -2, x = 2$~~ not in domain

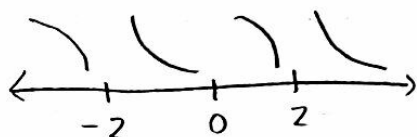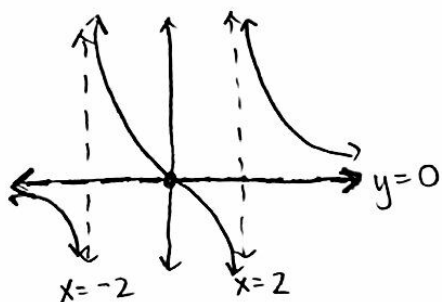$y'' \xleftarrow{\;\; (-) \; \vdots \; (+) \; (-) \; \vdots \; (+) \;\;}$

     -2 0 2

CU $(-2, 0) \cup (2, \infty)$

CD $(-\infty, -2) \cup (0, 2)$

IPs: $(0, 0)$

⑤

$x = -2$ $x = 2$

How to Sketch a Graph

① Find the domain. (Precalc)

② Find the intercepts, asymptotes.
     (Precalc)   (Lessons 4, 22)

③ Find CV's, intervals of incr/decr, relative extrema   (Lessons 17, 18)

④ Find possible IPs, intervals of CU/CD, and IPs   (Lesson 19)

⑤ Sketch graph
    1st: asymptotes, points ①, ②
    2nd: shape of graph ③, ④

     -2 0 2